

(3 Hours)

[Total Marks : 80

- N.B.** (1) Question No. 1 is compulsory.
 (2) Attempt any **three** questions out of the remaining **five** questions.
 (3) Assume suitable **data** wherever **necessary**.

1. (a) Determine the fundamental period of the following signals :—

20

$$(i) \quad x(t) = \cos \frac{\pi}{3} t + \sin \frac{\pi}{4} t$$

$$(ii) \quad x[n] = \cos^2 \frac{\pi}{8} n$$

- (b) State and prove Time Shifting and Time Scaling property of continuous time Fourier Transform.
 (c) For the following system, determine whether it is. (i) memory less, (ii) causal, (iii) linear, (iv) time-invariant $y[n] = x[n^2]$
 (d) Find out even and odd component of the following two signals :
 (i) $x(t) = t^3 + 3t$
 (ii) $x[n] = \cos n + \sin n + \cos(n) \sin(n)$
 (e) Determine whether the signals are power or energy signals. Calculate energy / power accordingly :
 (i) $x(t) = 0.9 e^{-3t} u(t)$
 (ii) $x[n] = u[n]$

2. (a) Find the inverse Laplace Transform of $\frac{s-2}{s(s+1)^3}$

5

(b) Let $x(t) = 1 \dots \dots 0 \leq t \leq 2T$ and ; $h(t) = e^{-at} \dots \dots 0 \leq t \leq T$. Compute $y(t)$ using graphical convolution approach. 10

(c) State and discuss the properties of the region of convergence for Z Transform. 5

3. (a) An LTI system is characterized by the system function :

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$$H(z) = \frac{z}{\left(z - \frac{1}{4}\right) \left(z + \frac{1}{4}\right) \left(z - \frac{1}{2}\right)}$$

Write down possible ROCs. For different possible ROCs, determine causality and stability and impulse response of the system.

(b) Calculate Z transform of the following signals :

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$$(i) \quad x[n] = n \left(-\frac{1}{4}\right)^n u[n] * \left(-\frac{1}{6}\right)^{-n} u[-n]$$

$$(ii) \quad x[n] = u[n-6] - u[n-10]$$

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4. (a) For the periodic signal $x(t) = e^{-t}$ with a fundamental period $T_0 = 1$ second. Find the exponential form of Fourier Series. Also plot the Fourier spectrum (Magnitude and phase spectrum) 10

(b) Consider a continuous time LTI system described by $\frac{dy(t)}{dt} + 2y(t) = x(t)$. Using the Fourier transform, find out output to each of the following input signals. 10

(i) $x(t) = e^{-t}u(t)$ (ii) $x(t) = u(t)$

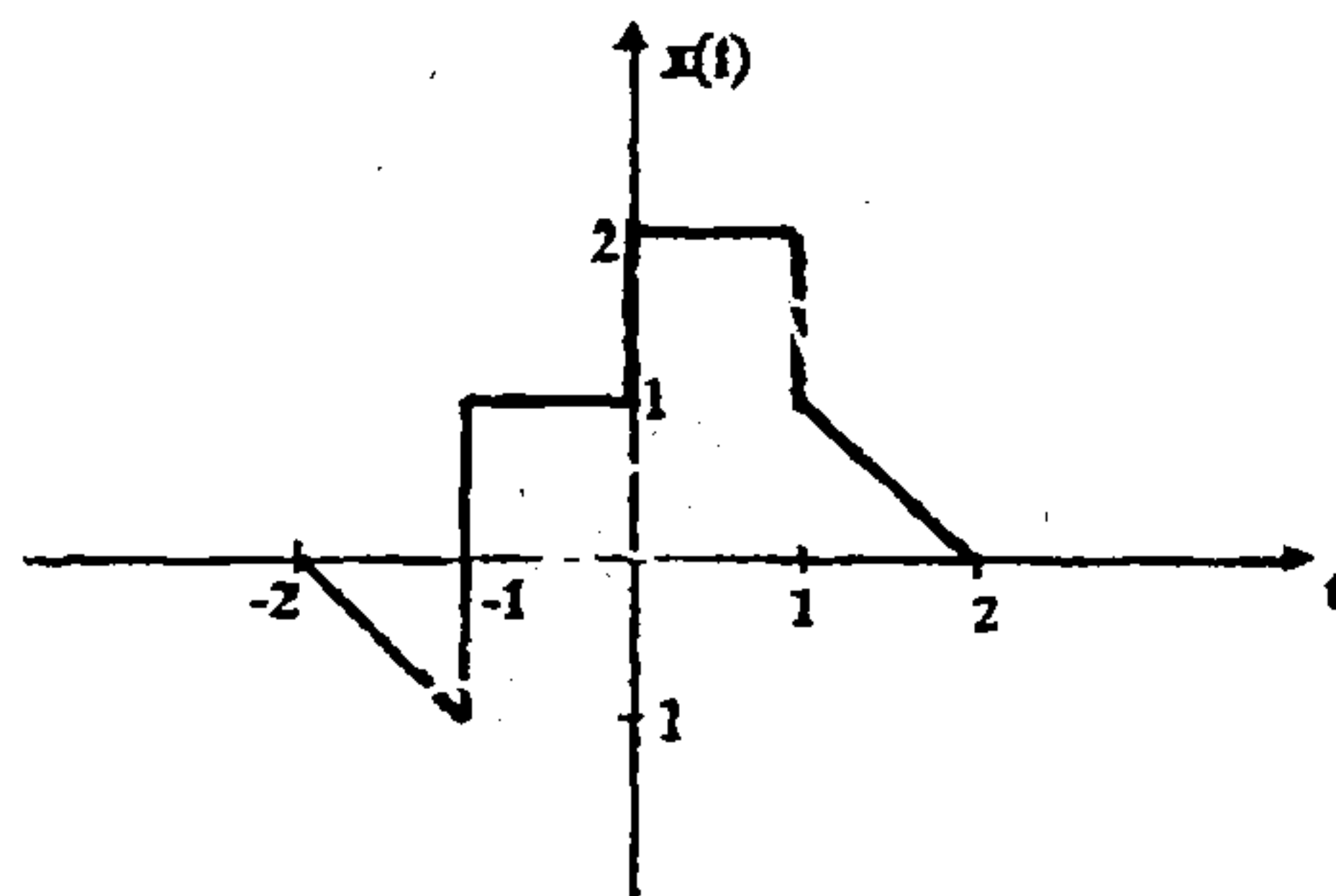
5. (a) Convolve $x[n] = \left(\frac{1}{3}\right)^n u[n]$ with $h[n] = \left(\frac{1}{2}\right)^n u[n]$ using convolution sum formula and verify your answer using z transform. 10

(b) Explain Gibb's phenomenon. Also explain conditions necessary for the convergence of Fourier Series. 5

(c) A system is described by the following difference equation. Find out its transfer function $H(z)$. 5

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n] + \frac{1}{2}x[n-1]$$

6. (a) For the signal $x(t)$ depicted in the figure given below, sketch the signals : 10



- (i) $x(-t)$
- (ii) $x(t+6)$
- (iii) $x(3t)$
- (iv) $x(t/2)$

(b) For the periodic signal $x[n]$ given below, find out Fourier Series coefficient : 10

$$x[n] = 1 + \sin\left(\frac{2\pi}{N}n\right) + 3\cos\left[\frac{2\pi}{N}n\right] + \cos\left(\frac{4\pi}{N}n + \frac{\pi}{2}\right)$$